Technoscience Journal for Community Development in Africa 2:1 (2021) 121–129
ISSN 2659-0573

Research Article

Comparative studies of the alpha decay half-lives of $^{197-220}$Fr isotopes using the modified Gamow-like model and other empirical formulas

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Abstract: Alpha decay is an important decay mode that provides information about the structure and stability of heavy and superheavy nuclei. In this study, Gamow-like model (GLM), modified Gamow-like model (MGLM1), and four empirical formulas were employed to study the alpha decay half-lives of $^{197-220}$Fr isotopes. The results obtained were compared with available experimental data and other theoretical calculations. The four empirical formulas used in the study are the Akrawy, new Ren B, AKRE, and Horoi formulas. Among the empirical formulas, the results of the calculations suggest that the new Ren B formula is the most suitable for the calculation of alpha-decay half-lives of the Fr isotopes. Calculated standard deviations showed that the MGLM1 gave a better description of the half-lives than the GLM model. New parameter values were obtained for the $^{197-220}$Fr isotopes using the modified Gamow-like model (termed MGLM2). The MGLM2 model gave a lower deviation from experimental values when compared with the previous theoretical calculations using the temperature-dependent proximity potential model.

Keywords: Alpha-decay; Gamow-like model; modified Gamow-like model; new ren B

1. Introduction

Alpha decay is one of the decay modes for heavy nuclei. It was first discovered in 1899 by Rutherford (Zdeb, Warda, & Pomorski, 2013). This decay mode has been successfully described by quantum theory (Hassanabadi et al., 2013), and it provides information about the nuclear structure and stability of heavy and superheavy nuclei (Cheng et al., 2019). It gives useful insights for the identification of new heavy and super heavy nuclei (SHN) (Alsaif, Radiman, & Ahmed, 2017), and in the study of nuclear force (Santhosh et al., 2020). There are several theoretical investigations on α-decay half-lives, and several theoretical models such as fission-like model (Wang et al., 2010), generalized liquid drop model (Royer & Moustabchir, 2001; Xiaojun et al., 2014; Royer and Zhang, 2008), modified generalized liquid drop model (Santhosh et al. 2018, 2020; Santhosh and Jose 2019), the effective liquid drop model (Cui et al. 2018), and the preformed cluster model (Gupta & Greiner, 1994; Singh, Patra, & Gupta, 2010), have been used to study the α-decay half-lives. Different interaction potentials ranging from phenomenological potentials (Santhosh, Sahadevan, and Biju, 2009; Santhosh et al., 2012; Zanganah et al., 2020) to microscopic potentials have been employed in the study of α-decay half-lives. Also, various empirical formulas have been used to calculate α-decay half-lives of many isotopes, examples include Royer formula (Royer, 2010; Royer, Schreiber, and Saulnier, 2011); universal decay law (Qi, Xu, Liotta, Wyss, et al., 2009; Qi, Xu, Liotta, and Wyss, 2009); Ren formula (Ren, Xu, and Wang, 2004), and its modified versions new Ren A and new Ren B formulas (Akrawy et al., 2019); Akrawy formula (Akrawy and Ahmed, 2018); scaling law of Brown; scaling law of Horoi (Horoi, 2004), and AKRE formula (Akrawy & Poenaru 2017). Unlike the Geiger-Nuttall law, the listed empirical formulas include contributions of nuclear asymmetry

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term $(I = (N - Z)/A)$. Moreover, the Akrawy and new Ren B formulas also include the angular momentum and isobaric asymmetry factors. The angular momentum represents the centrifugal potential contribution, which is known to increase the height of the potential barrier for odd systems.

The first decay law to describe α-decay half-life was proposed by Geiger and Nuttall in 1911 and was given theoretical explanation by Gamow in 1928 (Gamow, 1928). The Gamow theory explained that the α-decay was due to the quantum mechanical tunneling of a charged α particle through the nuclear Coulomb barrier (Zdeb et al., 2013). New phenomenological models which are based on the Gamow theory `had been introduced. For example, Zdeb et al. (2013) introduced a Gamow-like model (GLM) to compute the α-decay half-lives of various isotopes, with $Z \geq 84$ and $N \geq 104$. In the GLM, square well potential is chosen as the nuclear potential, Coulomb potential is taken to be a uniformly charged sphere, nuclear radius constant as an adjustable parameter, while centrifugal potential is ignored (Zdeb, Warda, & Pomorski, 2013). The GLM includes hindrance factor, the effect of an odd-proton, and/or an odd-neutron. The hindrance factor is unaccounted for in the Gamow theory. The GLM gave very good descriptions of the α-decay half-lives of 298 α-emitters, and the better results were attributable to the inclusion of hindrance factor (especially for non-even nuclei) and nuclear radius constant. The model, however, ignored the centrifugal potential.

Recently, Cheng et al. (2019), introduced a modified form of the Gamow-like model (MGLM) to compute the α-decay half-lives of nuclei with $Z > 51$. The alpha-daughter nucleus potential in the MGLM includes the Hulthen type of screened electrostatic Coulomb potential and the centrifugal potential. Unlike the GLM, the MGLM includes the effect of the centrifugal potential and electrostatic shielding. The model contains, as adjustable parameters, the radius constant, a parameter that is related to the screened electrostatic barrier, and the hindrance factor necessary for odd-odd and odd-A nuclei. The model, therefore, gives better descriptions of the alpha decay half-lives of nuclei. These two models (GLM & MGLM) are used to calculate the α-decay half-lives of the Fr isotopes in this work.

Francium (Fr) is known (to date) to have 36 isotopes, and none of these isotopes is stable (Zanganah et al., 2020). The most stable of the Fr isotopes is ^{223}Fr, and it has a half-life of 22 minutes. With a half-life of $0.12\ \mu s$, ^{215}Fr is the most unstable ground state isotope of Francium (Zanganah et al., 2020). Uusitalo et al. (2013) reported the measurement of the α-decay half-lives of ^{198,199}Fr. The theoretical study of the α-decay half-lives of $^{197-220}$Fr isotopes have also been carried out using a temperature-dependent proximity potential model. In the present study, the α-decay half-lives of the Francium isotopes are calculated using the GLM and MGLM models. The results are then compared to four empirical formulas to study the performance of these two models. The four empirical formulas are the Akrawy, new Ren B, AKRE, and Horoi formulas. To put on the same footing the comparison of the MGLM model with the temperature-dependent proximity potential model (CPPMT) employed in the theoretical calculations of Zanganah et al. (2020)., there is the need to obtain new values for the adjustable parameters using the experimental values of the Francium isotopes. This is required because the CPPMT model made use of temperature values calculated for each of the Francium isotopes. The three adjustable parameters viz. the screening parameter α in the Hulthen potential, the radius constant r_0, and the hindrance factor h, are computed here by fitting the MGLM model with the experimental α-decay half-lives of the $^{197-220}$Fr isotopes. This is then termed as MGLM2. The article is organized as follows. The modified Gamow-like model and the four empirical formulas employed for the calculation of the α-decay half-lives are summarised in Section 2. In Section 3, the results of the calculations are presented and discussed, while the conclusion is given in Section 4.

2. Methodology

2.1. Modified Gamow-like model

Here, a summary of the modified Gamow-like model (MGLM) is presented. The Gamow-like model (GLM) used in this work is as described by Zdeb, Warda, and Pomorski (2013). In the modified Gamow-like model, the interaction potential between the alpha particle and daughter nucleus is given by (Cheng et al., 2019; Yahya, 2020):

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq R \\ V_H(r) + V_\ell(r), & r \geq R, \end{cases} \tag{1}$$

where the Hulthen type of screened electrostatic Coulomb potential is given as

$$V_H(r) = \frac{a Z_1 Z_2 e^2}{e^{ar} - 1}, \tag{2}$$

and the centrifugal potential is

$$V_\ell(r) = \frac{\left(\ell+\frac{1}{2}\right)^2 \hbar^2}{2\mu r^2}, \quad (3)$$

V_0 is the depth of the square well, Z_1 and Z_2 are the atomic numbers of the α particle and daughter nucleus, respectively, ℓ is the orbital angular momentum that the α particle takes away, and α is the screening parameter. The radius of the spherical square well is calculated by summing the radii of both the daughter nucleus (A_2) and the α particle (A_1) using:

$$R = r_0\left(A_1^{1/3} + A_2^{1/3}\right). \quad (4)$$

Here r_0 is a constant, an adjustable parameter. The α decay half-life is calculated using (Cheng et al. 2019; Zdeb, Warda, & Pomorski 2013; Yahya, 2020):

$$T_{1/2} = \frac{\ln 2}{\lambda} 10^h, \quad (5)$$

where h is the decay hindrance factor due to the effect of an odd-neutron and/or an odd-proton. The value is zero for even-even nuclei. The values of the three parameters (a, r_0, and h) in the model were determined by Cheng et al. (2019) to be:

$$a = 7.8 \times 10^{-4},\ r_0 = 1.14 \text{ fm},\ h = 0.3455. \quad (6)$$

For odd-odd nuclei, $h_{np} = 2h$. The decay constant λ is computed using:

$$\lambda = \nu S P, \quad (7)$$

where S is the alpha particle preformation probability at the nuclear surface. The best-fitting results were obtained by setting $S = 1$ in both the GLM and MGLM models (Cheng et al. 2019; Zdeb, Warda, and Pomorski 2013). The penetration probability P is given by

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \sqrt{2\mu\left(V(r) - E_k\right)}\ dr\right], \quad (8)$$

$\mu = \frac{A_1 A_2}{A1+A2}$ MeV denotes the reduced mass of the daughter nucleus and the α particle, and the kinetic energy of the emitted α particle is denoted by $E_k = Q_\alpha (A-4)/A$.

The classical turning point b is obtained through the condition $V(b) = E_k$. In this model, the frequency of assault on the potential barrier and the radius of the parent nucleus are calculated using

$$\nu = \frac{(G+3/2)\hbar}{1.2\pi\mu R_0^2}, \quad (9)$$

and

$$R_0 = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3} \quad (10)$$

respectively. The global quantum number G is determined via the Wildermuth quantum rule. For alpha decay, G is calculated using:

$$G = \begin{cases} 22 & N > 126 \\ 20 & 82 < N \le 126, \\ 18 & N \le 82 \end{cases} \quad (11)$$

where N is the neutron number.

2.2. Empirical formulas

Here, we give brief descriptions of the four empirical formulas used in this study. Many empirical formulas have been developed after the Geiger-Nuttall law. The new formulas include additional terms, such as nuclear isospin asymmetry and orbital angular momentum, to improve the results of the alpha decay half-lives. The four formulas used in this work are the Akrawy, new Ren B, AKRE, and Horoi formulas. The computed α-decay half-lives using the GLM and MGLM will be compared with the empirical formulas.

2.2.1. AKRE

Akrawy and Poenaru (2017) modified the Royer formula (Royer, 2010; Royer, Schreiber, & Saulnier, 2011) for α-decay half-lives by including the nuclear asymmetry term $(I = (N-Z)/A)$. The formula is given as:

$$\log_{10}\left(T_{1/2}^{\text{AKRE}}\right) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2} + dI + eI^2, \quad (12)$$

where the parameters a, b, c, d, and e, obtained by fitting experimental data, are given in Table 1 (Akrawy et al., 2018).

2.2.2. Akrawy formula

Akrawy and Ahmed (2018) presented a new formula to calculate the α-decay half-lives of nuclei. They introduced three different physical terms viz. the orbital angular momentum and isobaric asymmetry factors based on the Royer (Royer, 2010) and Denisov and Khudenko (Denisov & Khudenko, 2009) formulas. The Akrawy formula is given by:

$$\log_{10}\left(T_{\frac{1}{2}}^{\text{Akrawy}}\right) = a + bA^{\frac{1}{6}}Z^{\frac{1}{2}} + cZQ^{-\frac{1}{2}} + d\sqrt{\ell(\ell+1)}Q^{-1} + eI + f\mu I^2[\ell(\ell+1)]^{1/4}, \quad (13)$$

where A, Z, I, Q, μ, ℓ are the mass number, atomic number, nuclear asymmetry term (I), decay energy, reduced mass, and orbital angular momentum quantum number, respectively. The coefficients a, b, c, d, e, and f, were obtained through the least square fit procedure. They are given in Table 2.

2.2.3. Scaling law of Horoi

Horoi (2004) presented an empirical formula to determine the half-lives of both alpha and cluster decays and is given by:

$$\log_{10}\left[T_{\frac{1}{2}}^{\text{SLH}}(s)\right] = (a_1\mu^x + b_1)\left[(Z_1Z_2)^y/\sqrt{Q} - 7\right] + (a_2\mu^x + b_2), \quad (14)$$

where μ is the reduced mass and the parameters a_1, b_1, a_2, b_2, x, y that contain information on the dynamics of the decay are given by $9.1, -10.2, 7.39, -23.2, 0.416$, and 0.613, respectively (Hosseini, Hassanabadi, & Zarrinkamar 2018; Hosseini, Hassanabadi, & Sobhani, 2017).

2.2.4. New Ren B formula

The modified forms of the Ren A and Ren B formulas (Ren, Xu, & Wang, 2004) that included the nuclear isospin asymmetry term were given in Ref. (Akrawy et al., 2019) as New Ren A (with five free parameters) and New Ren B (with six free parameters), respectively. The New Ren B (NRB) formula included both nuclear isospin asymmetry and angular momentum. The angular momentum represents the centrifugal potential contribution. It is known that the centrifugal potential increases the height of the potential barrier for odd-odd, odd-even, and even-odd nuclei. However, the value of the centrifugal term is zero for even-even nuclei. It should be noted that the Ren formulas are generalizations of the Viola-Seaborg formula, which itself depends on the Geiger-Nuttall law. The formula is given as:

$$\log_{10}\left[T_{\frac{1}{2}}^{\text{NRB}}(s)\right] = a\sqrt{\mu}Z_1Z_2\sqrt{Q} + b\sqrt{\mu Z_1Z_2} + c + dI + eI^2 + f[\ell(\ell+1)], \quad (15)$$

where Z_1 and Z_2 are the atomic numbers of the cluster and daughter nuclei, respectively, I is the nuclear isospin $(I = (N-Z)/A)$, $\mu = A_1A_2/(A_1 + A_2)$ is the reduced mass, and the angular momentum ℓ are obtained from the selection rule given by (Qi et al., 2012; Denisov, Davidovskaya, & Sedykh, 2015; Akrawy et al., 2019):

$$\ell = \begin{cases} \Delta_j & \text{for even } \Delta_j \text{ and } \pi_d = \pi_p \\ \Delta_j + 1 & \text{for odd } \Delta_j \text{ and } \pi_d = \pi_p \\ \Delta_j & \text{for odd } \Delta_j \text{ and } \pi_d \neq \pi_p \\ \Delta_j + 1 & \text{for even } \Delta_j \text{ and } \pi_d \neq \pi_p \end{cases} \quad (16)$$

Here $\Delta_j = |j_p - j_d|$, where j_d, π_d, j_p, π_p are the spin and parity values of the daughter and parent nuclei, respectively. The values of the six parameters a, b, c, d, e and f are given in Table 3.

3. Results and discussions

The results of the computation of the α-decay half-lives for the 24 Francium isotopes $(^{197-220}\mathrm{Fr})$ with $Z = 87$ are presented here. The calculations have been carried out using the Gamow-like model (GLM), the modified Gamow-like model using the parameters of Cheng et al. (2019) (termed MGLM1), and using new parameters determined through a least-square fit (termed MGLM2), and four empirical formulas viz. the Akrawy, new Ren B, AKRE, and Horoi formulas. The three parameters in the modified Gamow-like model have been determined through a fit to the experimental half-lives. The calculated values of the three parameters in the modified Gamow-like model are

$$r_0 = 1.181 \text{ fm}, \; h = 0.2912, \; a = 1.000 \times 10^{-7} \quad (17)$$

for the Fr isotopes, with a root mean square standard deviation value of 0.3294. As noted by Cheng et al. (2019), even though the value of the parameter a is small, it affects the classical turning point b, which in turn affects the accuracy of the α-decay half-lives. The experimental data used in the study have been taken from the NUBASE2016 (Audi et al., 2017; Wang et al., 2017; Huang et al., 2017).

Table 4 shows the calculated alpha-decay half-lives for the $(^{197-220}\mathrm{Fr})$ isotopes. The first three columns show the mass number (A), the experimental Q_α values, and the experimental α-decay half-lives (Expt.)

Table 1: Coefficients of the AKRE formula.

Set	a	b	c	d	e
even-even	-26.32279	-1.15985	1.59227	12.06060	-41.66328
even-odd	-24.40718	-1.2320	1.65492	-31.86294	159.77682
odd-even	-31.79248	-1.07636	1.75354	-2.22627	0.39378
odd-odd	-26.27896	-1.20130	1.65906	-10.08411	67.59728

Table 2: Coefficients of the Akrawy formula.

Set	a	b	c	d	e	f
even-even	-25.3860	-1.1561	1.5857	0.00	-0.2050	0.00
even-odd	-29.0583	-1.0612	1.6477	0.0426	-1.3405	6.8970
odd-even	-31.6038	-1.0003	1.6943	2.6263	-3.5278	-0.0039
odd-odd	-28.2580	-1.0811	1.6290	0.8047	1.8276	3.6070

Table 3: Coefficients of the New Ren B formula.

Set	a	b	c	d	e	f
even-even	0.41107	-1.44914	-14.87085	13.38618	-61.47107	0.0000
even-odd	0.44145	-1.42068	-16.59713	-27.68464	91.70405	0.07947
odd-even	0.44660	-1.32208	-21.09761	-1.64226	-17.02692	0.07767
odd-odd	0.43323	-1.40527	-17.13866	-7.66291	22.26925	0.06902

$(\log[T_{1/2}(s)])$, respectively. In the last seven columns of the Table, the computed α-decay half-lives using the GLM, MGLM1, MGLM2, Akrawy, New Ren B (NRB), AKRE, and Horoi formulas are shown. A physical inspection of the Table shows that the MGLM2 and New Ren B formula give better descriptions of the alpha decay half-lives than the remaining models.

Moreover, in order to quantitatively compare the agreement between the experimental half-lives and the theoretically calculated half-lives using the various models, the root mean square standard deviation (σ) has been computed. The following formula has been used to calculate σ:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(\log_{10}T_{1/2,i}^{\text{Theory}} - \log_{10}T_{1/2,i}^{\text{Expt}}\right)^2\right]}. \quad (18)$$

Here $T_{1/2,i}^{\text{Expt}}$ are the experimental half-lives and $T_{1/2,i}^{\text{Theory}}$ denote the half-lives obtained using the theoretical models. The results of the standard deviation (σ) calculations using the different models are displayed in Table 5. The second to ninth columns of the Table show, respectively, the computed standard deviations using GLM, MGLM1, MGLM2, Akrawy, New Ren B (NRB), AKRE, Horoi formulas, and the temperature-dependent proximity potential model of Zanganah et al. (2020). Zanganah et al. (2020) studied the α-decay half-lives of the Fr isotopes using temperature-independent and temperature-dependent proximity potential models. They employed the use of the prox 88, prox 2010, and prox Zheng proximity potential models. The lowest standard deviation was obtained with the use of the temperature-dependent proximity potential model (TDPPM) using the prox 88 version with a standard deviation of 0.3396. This is the standard deviation shown in the ninth column of Table 5. The use of temperature-dependent proximity potential models has been shown, in recent times, to give very good descriptions of α-decay half-lives (Yahya, 2020). The calculated standard deviations obtained for the GLM, MGLM1, MGLM2, Akrawy, NRB, AKRE, and Horoi formulas are shown in Table 5 to be $0.5575, 0.4462, 0.3294, 0.4482, 0.3039, 0.5564$ and 0.6425, respectively. The results show that the MGLM1 model gives a lower standard deviation than

Table 4: Calculated α-decay half-lives $(log[T_{1/2}(s)])$, of $^{197-220}Fr(Z = 87)$ using GLM, MGLM1, MGLM2, Akrawy, New Ren B (NRB), AKRE, and Horoi formulas

A	Q_α (MeV)	Expt.	GLM	MGLM1	MGLM2	Akrawy	NRB	AKRE	Horoi
197	7.9000	-2.6326	-1.8729	-1.9431	-2.2199	-2.0778	-2.1729	-1.9868	-2.6063
198	7.8690	-1.8239	-1.5822	-1.5233	-1.8532	-1.8591	-2.0369	-2.1052	-2.5093
199	7.8170	-2.1805	-1.6577	-1.7294	-2.0028	-1.8693	-1.9389	-1.7591	-2.3461
200	7.6220	-1.3233	-0.8427	-0.7923	-1.1104	-1.0718	-1.2179	-1.2628	-1.7236
201	7.5190	-1.2020	-0.7416	-0.8245	-1.0830	-0.9030	-0.9346	-0.7473	-1.3843
202	7.3860	-0.4295	-0.0990	-0.0579	-0.3634	-0.2828	-0.3934	-0.4096	-0.9362
203	7.2750	-0.2596	0.0477	-0.0455	-0.2904	-0.0764	-0.0749	0.1199	-0.5525
204	7.1700	0.2430	0.6143	0.6459	0.3529	0.4721	0.3997	0.4178	-0.1812
205	7.0540	0.5821	0.7981	0.6945	0.4631	0.7066	0.7381	0.9416	0.2384
206	6.9230	1.2800	1.4816	1.5005	1.2236	1.3834	1.3570	1.4125	0.7247
207	6.8930	1.1703	1.3593	1.2477	1.0268	1.2854	1.3408	1.5518	0.8394
208	6.7850	1.7716	1.9713	1.9831	1.7155	1.9037	1.9152	2.0168	1.2539
209	6.7770	1.7033	1.7665	1.6492	1.4361	1.6991	1.7731	1.9910	1.2866
210	6.6720	2.2806	2.3780	2.6138	2.3495	3.2843	2.8018	2.5406	1.7000
211	6.6620	2.2695	2.1822	2.0588	1.8539	2.1218	2.2128	2.4392	1.7415
212	6.5290	3.0813	2.9209	3.1486	2.8952	3.9246	3.4200	3.2125	2.2803
213	6.9040	1.5333	1.1984	1.0926	0.8690	1.0488	1.1246	1.3394	0.8078
214	8.5890	-2.2857	-3.9557	-2.8153	-3.2040	-2.5495	-2.2347	-3.9557	-4.5763
215	9.5400	-7.0655	-6.5089	-6.5620	-6.9009	-7.3892	-7.4816	-7.3832	-6.9633
216	9.1740	-6.1549	-5.4662	-5.3952	-5.7790	-5.8120	-5.9116	-5.4762	-6.0874
217	8.4690	-4.7747	-3.9044	-3.9783	-4.2850	-4.5230	-4.5539	-4.4064	-4.2436
218	8.0140	-3.0000	-2.4113	-2.3693	-2.7119	-2.5645	-2.5326	-2.0626	-2.9259
219	7.4480	-1.6990	-0.8569	-0.9669	-1.2270	-1.2235	-1.1850	-0.9814	-1.1213
220	6.8000	1.4378	1.6782	1.7384	1.4642	2.6004	2.0308	2.3885	1.2156

Table 5: Calculated root means square standard deviation (σ) using the different models, compared with the previously obtained theoretical results (TDPPM) using the time-dependent proximity potential model (TDPPM)

Model	GLM	MGLM1	MGLM2	Akrawy	NRB	AKRE	Horoi	TDPPM (Zanganah et al., 2020)
σ	0.5575	0.4462	0.3294	0.4482	0.3039	0.5564	0.6425	0.3396

the GLM model. This can be attributed to the inclusion of the centrifugal potential and electrostatic shielding in the MGLM1 model. Also, the MGLM2 model gives a lower standard deviation compared to all the models in the Table except the new Ren B (NRB) formula. Among the empirical models, the New Ren B is the most suitable for the determination of the α-decay half-lives of the Fr isotopes, followed by the Akrawy formula. The success of both the new Ren B and Akrawy formulas can be attributed to the inclusion of both the nuclear isospin asymmetry and orbital angular momentum terms in the two models. The AKRE formula included the nuclear asymmetry term but it does not contain the contribution of the centrifugal potential. The scaling law of Horoi gives the highest standard deviation. In this work, we have obtained, using the

MGLM2, a lower standard deviation of 0.3294. This is an improvement over the results obtained by Zanganah et al., (2020) using temperature-dependent proximity potential models (which is 0.3396).

The calculated half-lives $\log[T_{1/2}(s)]$ using the seven theoretical models have been plotted against the neutron number in Figure 1. The experimental values are shown in black circles. The highest and lowest values of the half-lives agree with the values obtained in Ref. (Zanganah et al. 2020). The maximum value of the half-life is obtained for $N = 125$ $(^{212}\mathrm{Fr})$ while the minimum value is at $= 128$ $(^{215}\mathrm{Fr})$. This is due to the role of shell closure effects relative to the magicity (or near magicity) of the neutron number. A high alpha decay half-life indicates the magicity of the parent nucleus, while a low half-life indicates the magicity of the daughter nucleus. From Figure 1, the maximum half-life is obtained for the parent nucleus $^{212}\mathrm{Fr}\ (Z = 87, N = 125)$. This indicates the near magicity of the neutron number $N = 125$. The minimum half-life corresponds to the decay to the daughter nucleus $^{211}\mathrm{At}\ (Z = 85, N = 126)$. This indicates the neutron number magicity of the daughter nucleus. These observations show the role of neutron shell closure.

The difference between experimental and theoretical α-decay half-lives have also been calculated using the following equation:

$$\Delta T_{1/2} = \log_{10}\left[T_{1/2}^{\text{theor}}\right] - \log_{10}\left[T_{1/2}^{\text{expt}}\right] \qquad (19)$$

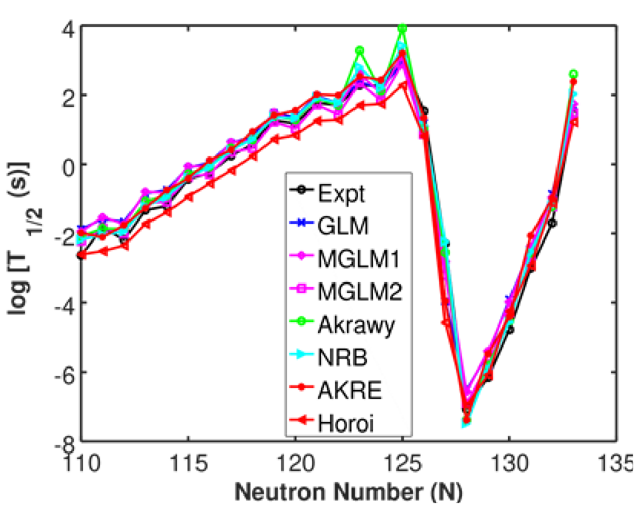

Figure 1: Comparison of the calculated α-decay half-lives of Fr isotopes between the various models and experiment

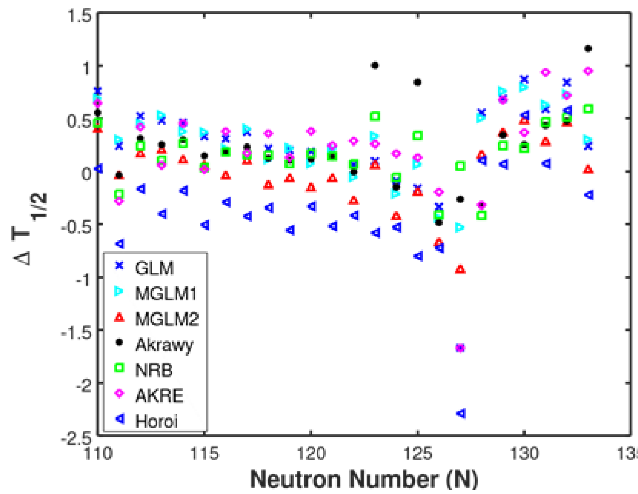

Figure 2: Plot of the ΔT against Neutron number (N) for the Fr isotopes using the different models

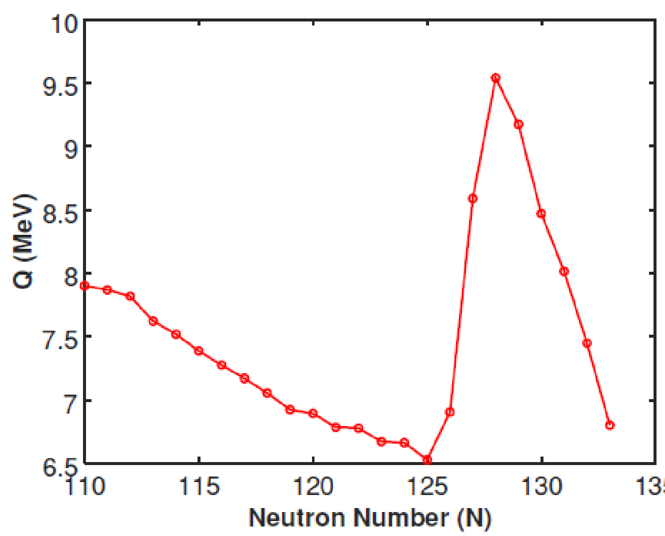

Figure 3: Plot of the Q_α against Neutron number (N) for the Fr isotopes

In Figure 2, $\Delta T_{1/2}$ has been plotted against neutron number for all the models used in this study. It can be observed that, barring few exceptions, most of the points are near zero and within ± 0.7. The plot of the experimental Q_α values against neutron number is shown in Figure 3. The minimum and maximum values of the Q_α are at $N = 125$ $(^{212}\mathrm{Fr})$ and $= 128$ $(^{215}\mathrm{Fr})$, respectively. These values correspond to the highest and lowest half-life values in Figure 1.

4. Conclusion

The study of α-decay half-lives of $^{197-220}\mathrm{Fr}$ isotopes have been carried out using the GLM, MGLM1, MGLM2 models, and four empirical formulas viz. the

Akrawy, new Ren B, AKRE, and Horoi formulas. The modified Gamow-like gives a better description of the half-lives than the Gamow-like model. We have obtained new parameters to be used in the modified Gamow-like model (termed MGLM2). The MGLM2 model gives the lowest standard deviation when compared with the results using the Gamow-like model (GLM) and modified Gamow-like model. Moreover, the MGLM2 model gives better results than the previously obtained results using the proximity potential models. Among all the empirical formulas used in the study, only the new Ren B formula gives a lower standard deviation than the MGLM2. All the models give α-decay half-lives which are in good agreement with the available experimental data, with the maximum standard deviation value less than 0.65. We conclude that, among the models used in this study, the MGLM2 and the new Ren B formula are the most suitable for calculating the α-decay half-lives for the Fr isotopes.

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